Logic for Computer Science. Knowledge Representation and Reasoning.

Lecture Notes

for

Computer Science Students
Faculty EAIiIB-IEiT AGH

Antoni Ligęza

Other support material:

http://home.agh.edu.pl/~ligeza

https://ai.ia.agh.edu.pl/pl:dydaktyka:logic:start#logic_for_computer_science2020

Multi-Valued Logics

In classical Propositional Calculus we have just 2 truth-vales; something can be True or False (technically: 1 and 0):

$$I\colon P \longrightarrow \{\mathbf{T}, \mathbf{F}\},$$

In Multi-Valued Logics there can be 3 (or more) values.

The first 3-valued logic was introduced by Jan Łukasiewicz in 1920.

$$I\colon P \longrightarrow \{0, \frac{1}{2}, 1\},$$

The meaning of $\frac{1}{2}$ is unknown; maybe becoming true or false in future.

The truth-tables are based on the following practical formulas:

- $I(\neg p) = 1 - I(p)$,
- $(p \wedge q) = \min(I(p), I(q))$,
- $I(p \vee q) = \max(I(p), I(q))$,
- $I(p \rightarrow q) = \min(1, 1 + I(q) - I(p))$.

In Relational Databases/SQL:

- *NULL* – unknown but existing value (date of birth),
- *NULL* – unknown, maybe not existing value (no. of telephone)
- *NULL* – value of an attribute not applicable to an object

AND	*TRUE*	*FALSE*	*NULL*
TRUE	*TRUE*	*FALSE*	*NULL*
FALSE	*FALSE*	*FALSE*	*FALSE*
NULL	*NULL*	*FALSE*	*NULL*

OR	*TRUE*	*FALSE*	*NULL*
TRUE	*TRUE*	*TRUE*	*TRUE*
FALSE	*TRUE*	*FALSE*	*NULL*
NULL	*TRUE*	*NULL*	*NULL*

NOT	*TRUE*	*FALSE*	*NULL*
TRUE	*FALSE*	*TRUE*	*NULL*

Fuzzy Logic

Let U be a classical set (a universe). Any subset X of U can be defined by the so-called characteristic function – a predicate – m:

$$m\colon U \to \{0, 1\}$$

so that $m(x) = 1$ iff $x \in X$.

A Fuzzy Set A defined in U is a pair $A = (U, \mu_A)$, where:

$$\mu_A\colon U \to [0, 1]$$

In classical Propositional Calculus we have just 2 truth-vales; something can be True or False (technically: 1 or 0):

$$I\colon P \longrightarrow \{\mathbf{T}, \mathbf{F}\},$$

In Fuzzy Logic there can infinitely many truth values belonging to the interval $[0, 1]$.

The notion of Fuzzy Sets and Fuzzy Logic was introduced by Lotfi Zadeh in 1965.

$$I\colon P \longrightarrow [0, 1],$$

The meaning of $I(p) = \alpha$ for $0 < \alpha < 1$ is that p is partially true.

The truth-tables are based on the following practical formulas:

- $I(\neg p) = 1 - I(p)$,
- $(p \wedge q) = \min(I(p), I(q))$,
- $I(p \vee q) = \max(I(p), I(q))$,
- $I(p \to q) = \min(1, 1 + I(q) - I(p))$.

Temporal Logics

In classical Propositional Calculus we have just 2 truth-vales; something can be True or False (technically: 1 or 0):

$$I \colon P \longrightarrow \{\mathbf{T}, \mathbf{F}\},$$

The logical value of any proposition $p \in P$ remains true or false over all the time of concern. In other words., the truth values of formulas does not change over time.

In dynamic systems the state – and so its description – does change over time.

In the simplest Propositional Temporal Logic there are two temporal operators introduced:

- $\Box$ – with the meaning always; all the time, and
- $\Diamond$ – with the meaning eventually; somewhere in the future.

So the intended meaning is:

- $\Box p$ – p holds all over the time (defining safety),
- $\Diamond p$ – p will eventually happen (defining liveness).

Possible temporal models:

- continuous vs. discrete time
- intervals vs. instants (time points),
- linear vs. branching time,
- symbolic sequential models vs. real time models.