Knowledge Representation and Reasoning
Introduction to Constraint Programming

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AGH Kraków
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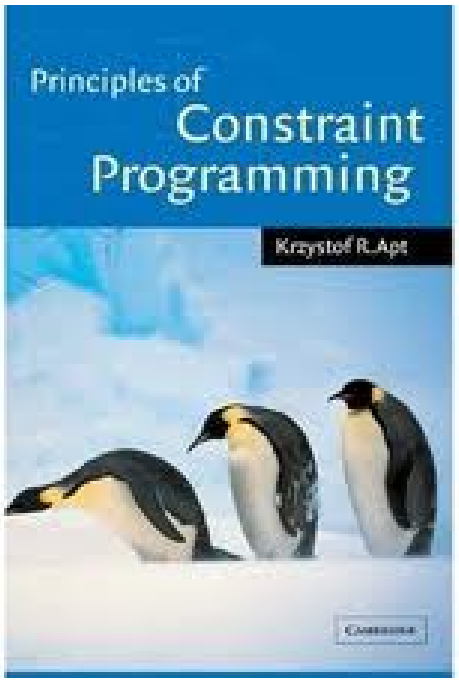
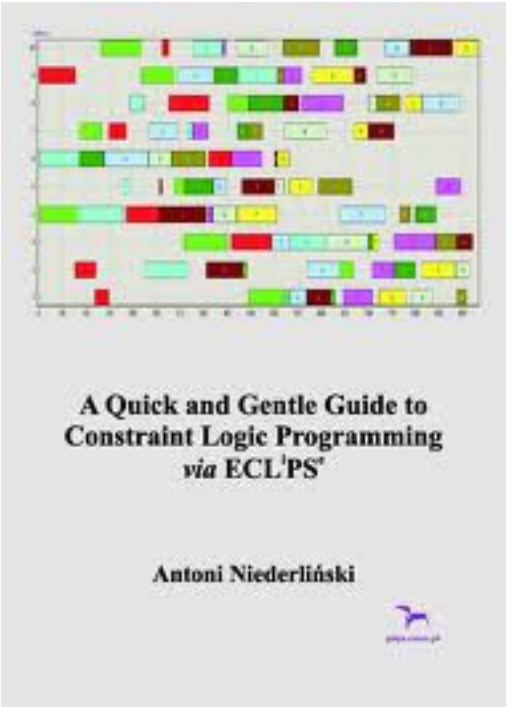
Outline

Presentation Outline

Some Books

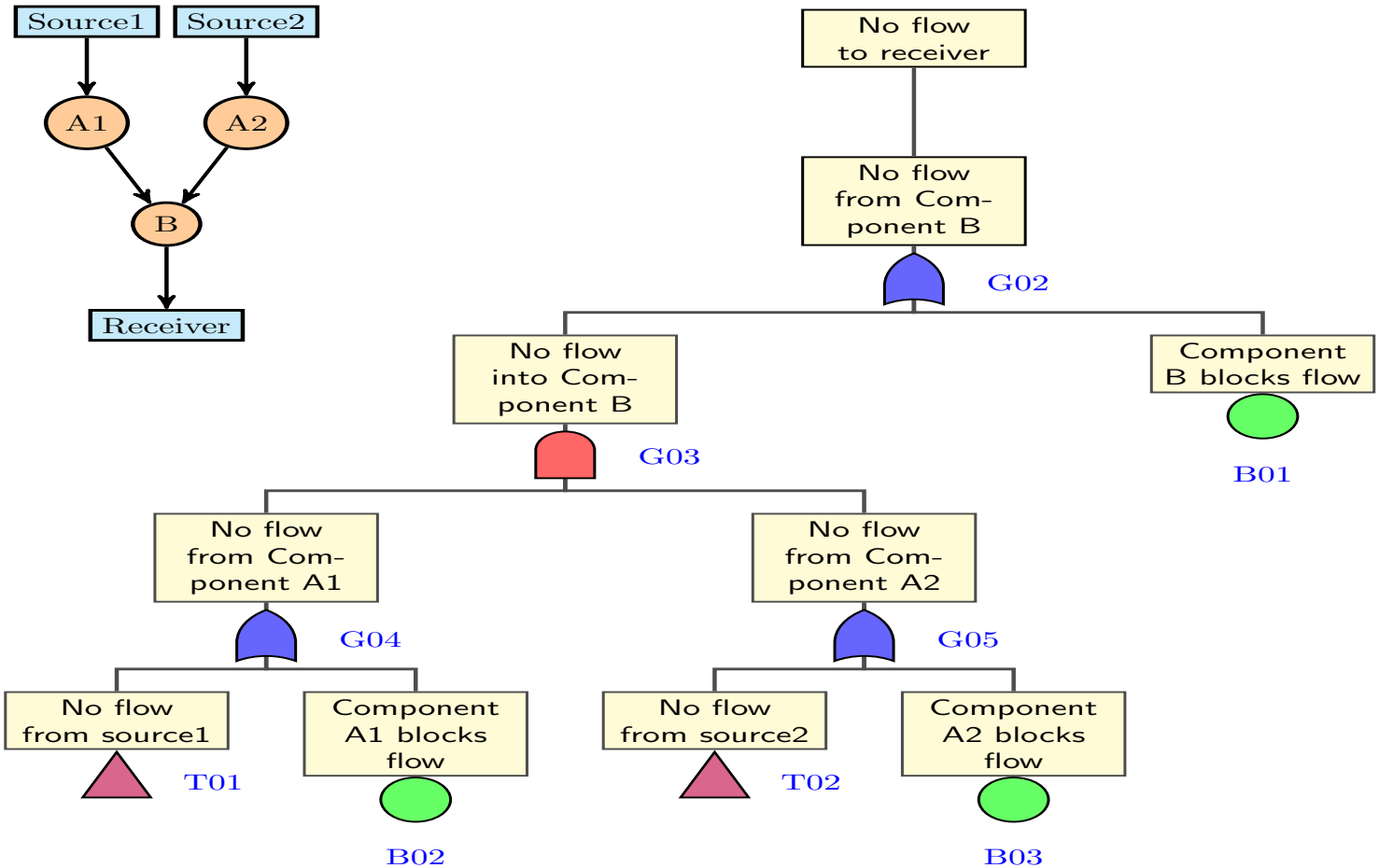

Principles of
Constraint
Programming
Krzysztof R. Apt


Handbook of Constraint Programming


A Quick and Gentle Guide to
Constraint Logic Programming
via ECLiPSe
Antoni Niederliński

Presentation Outline

An Example: System + Logic Model for Diagnostic Reasoning

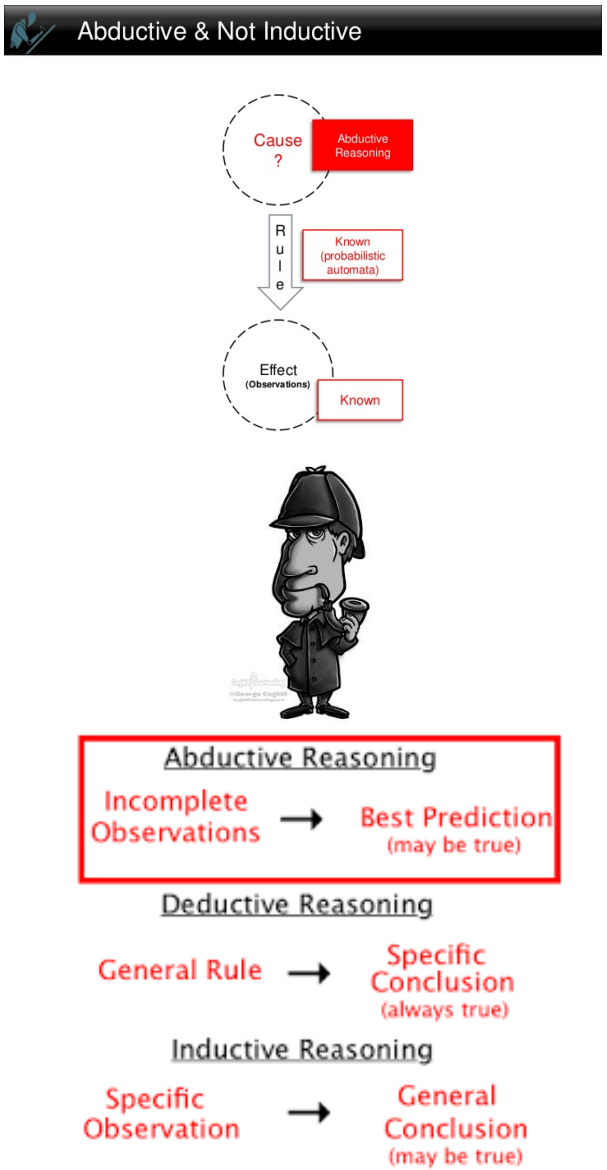


Presentation Outline

Abduction and Constraints: Introduction

Some loosely provocative questions and statements...

- abduction: what, why and where — what for?
- abduction: a method of logical inference (but invalid!),
- abduction vs. deduction,
- abduction: primary method used by Sherlock Holmes!
- abduction: inevitable ambiguity (potential/admissible solutions; many of them),
- abduction: more constraints — better abduction,
- abduction + constraints + SAT (minimal models).



Abduction and Constraints: Introduction

Abduction

- Abduction — principal way of problem solving — generation of hypotheses,
- Abduction — performed with backtracking search,
- Abduction — produces numerous, admissible solutions

Abduction: Logical model

$$\frac{\alpha \Longrightarrow \beta, \beta}{\alpha}$$

$$HYP^{+} \cup HYP^{-} \cup KB \models OBS^{+} \cup OBS^{-}$$

$$HYP^{+} \cup HYP^{-} \cup KB \cup OBS^{+} \cup OBS^{-} \not\models \bot$$

An intuitive example: find explanations for *wet_street*

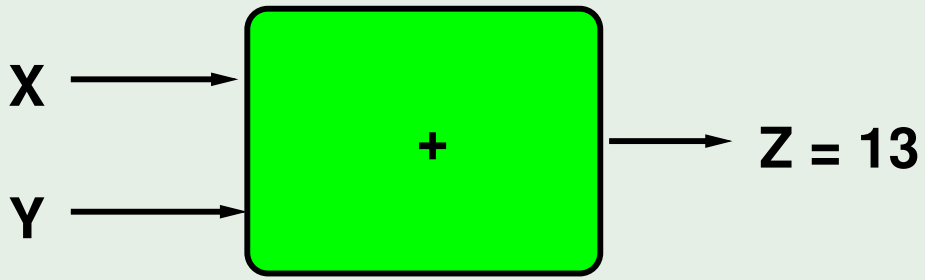
- *rain* $\longrightarrow$ *water*
- *sprinkler* $\longrightarrow$ *water*
- *snow* $\wedge$ *temperature* $\longrightarrow$ *water*
- *water* $\longrightarrow$ *wet_street*,
- *cleaning* $\longrightarrow$ *wet_street*
- *oil* $\longrightarrow$ *wet_street*

Presentation Outline

The role of constraints in abduction

Abductive problem without constraints

- X, Y, Z - variables, $X, Y \in \{0, 1, 2, \ldots, 9\}$, $Z \in \{0, 1, 2, \ldots, 18\}$
- system: $Z = X + Y$



- Observed: $Z = 13$
- Possible explanations:
  - $(X = 4$ and $Y = 9)$,
  - $(X = 5$ and $Y = 8)$,
  - ... ,
  - $(X = 9$ and $Y = 4)$.
- 6 admissible solutions.

The role of constraints in abduction

Abductive problem with constraints

- X, Y, Z - variables, $X, Y, Z \in \{0, 1, 2, \ldots, 9\}$,

$$Z = X + Y$$

- **Constraint:**

$$Y < X - 3$$

- Observed: $Z = 13$
- Possible explanations: $(X = 9$ and $Y = 4)$,
- 1 admissible solution.

Conclusion

- CONSTRAINTS can refine results of ABDUCTION; less models generated,
- propagation of CONSTRAINTS can reduce computational effort,
- ABDUCTION + CONSTRAINTS = CONSTRUCTIVE ABDUCTION

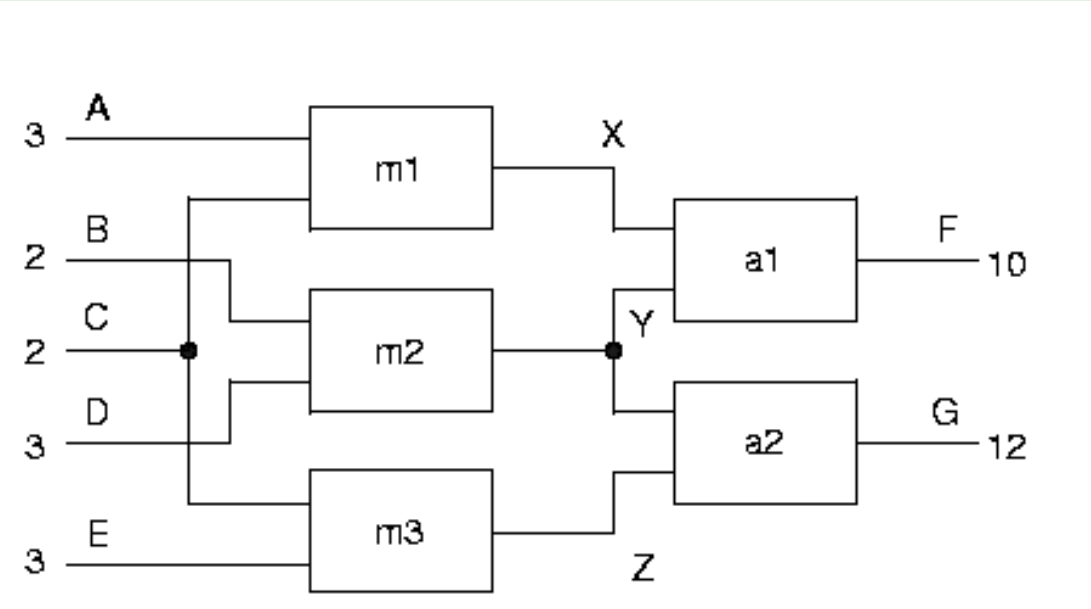
Presentation Outline

The example problem I

The Paradigm to be Explored Further on

ABDUCTION $\simeq$ DIAGNOSIS

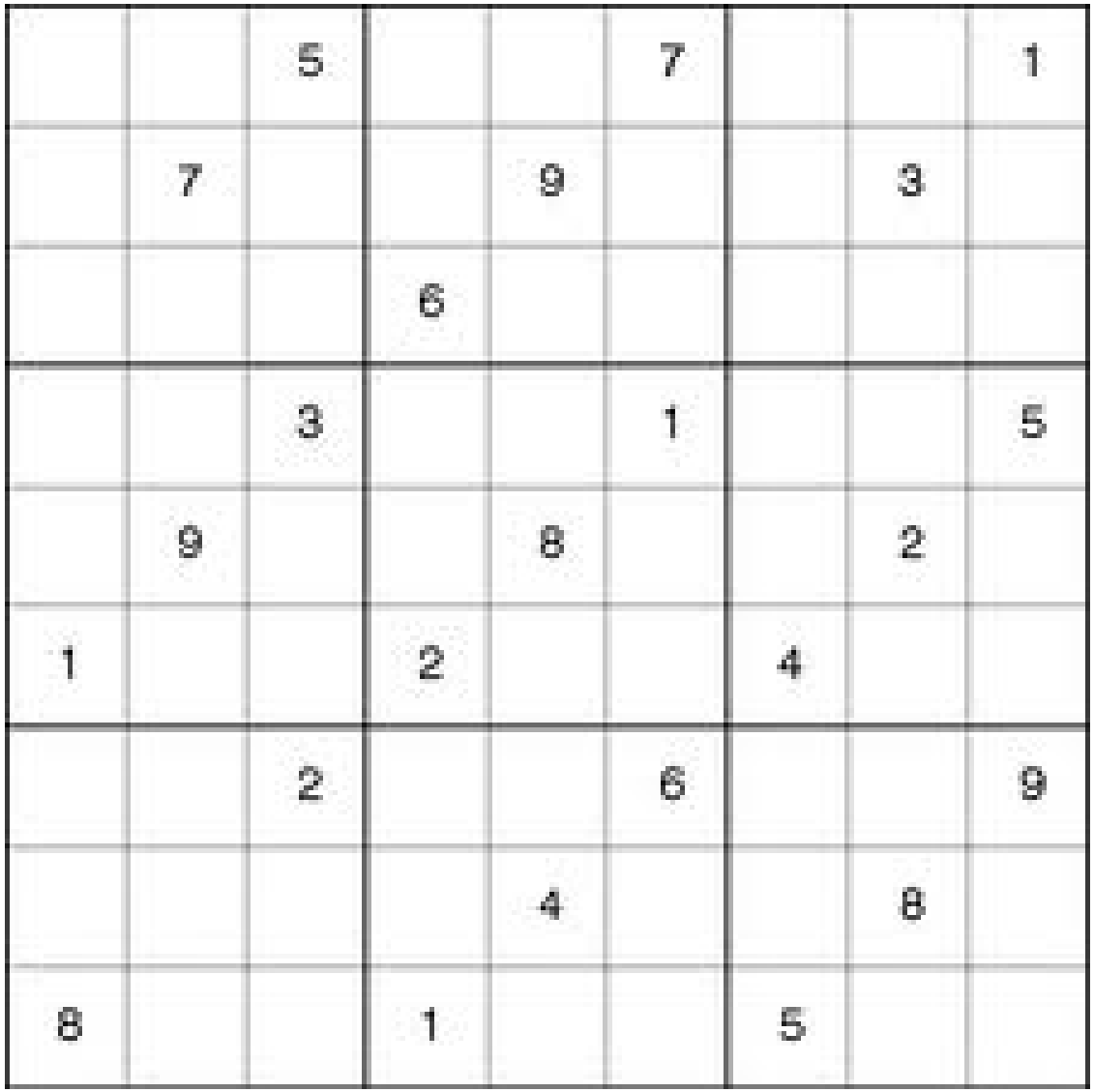
The Multiplier-Adder System




```
[...]
A #= 3,
B #= 2,
C #= 2,
D #= 3,
E #= 3,
F #= 10,
G #=12,
[...]
```

Presentation Outline

Constraint Satisfaction Problems



		5			7			1
	7			9			3	
			6					
		3			1			5
	9			8			2	
1			2			4		
		2			6			9
				4			8	
8			1			5		

Figure: Sudoku: An example Constraint Satisfaction Problem

Constraint Satisfaction Problems

Figure: An example Constraint Satisfaction Problem

Constraint Satisfaction Problems

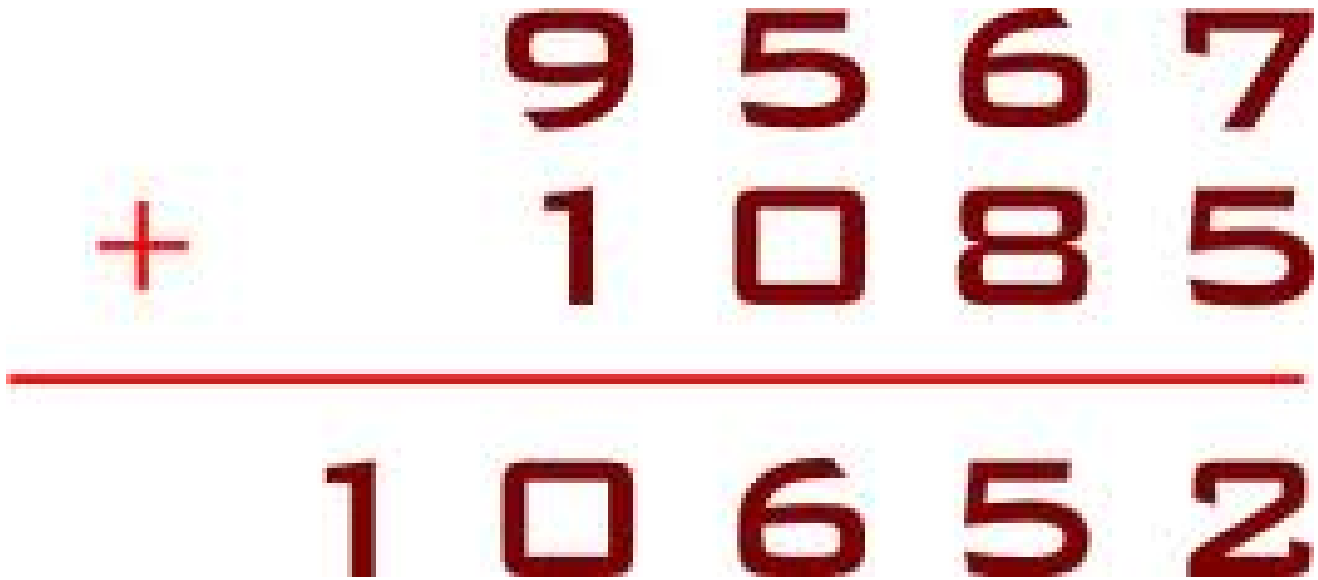


Figure: The unique solution of the example Constraint Satisfaction Problem

Constraint Satisfaction Problems

- Constraint (Logic) Programming:
  - simple, transparent statement; practical problem,
  - zero, one, or many solutions,
  - problem: combinatorial explosion.
- Decision factors in CP/CLP:
  - variable — which variable too choose,
  - value — which value to choose,
  - propagation — how to propagate constraints,
  - heuristics — what heuristics can be used.
- CP/CLP basic solution paradigm:
  - select a variable,
  - select a value,
  - propagate constraints,
  - if conflict — backtrack; if unique — return solution; else — loop.

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Constraint Satisfaction Problem

CSP statement

- $X = \{X_1, X_2, \ldots, X_k\}$ — a set of variables,
- $D = \{D_1, D_2, \ldots, D_k\}$ — their domains,
- $C = \{(S_i, R_i) : i = 1, 2, \ldots, n\}$ — constraints,
  - S_i — scope — a selection of variables,
  - R_i — relation defined over Cartesian Product of domains appropriate for the scope variables,

CSP solution

A solution to CSP given by (X, D, C) is any assignment of values to variables of X of the form

$$\{X_1 = d_1, X_2 = d_2, \ldots, X_k = d_k\},$$

such that $d_i \in D_i$, and for any constraint in $(S_i, R_i) \in C$, R_i is satisfied by the appropriate projection of the solution vector $(d_1, d_2, \ldots, d_k)$ over variables of S_i.

CP/CSP — Characteristics

CP vs. OPT

- CSP: first solution counts,
- OPT:best solution counts.

Binary vs. finite domains; SAT

- SAT: binary domains (0 or 1; true or false),
- CSP: finite discrete domains.

CSP: Problems

- large number of variables,
- large domains,
- numerous constraints,
- different types of constraints,
- unpredictable, irregular, hard to trace.

Presentation Outline

Example: SEND+MORE=MONEY

Figure: An example Constraint Satisfaction Problem

- 8 variables: `S, E, N, D, M, O, R, Y`,
- 10-values in each domain,
- alldifferent(S, E, N, D, M, O, R, Y),
- basic search-space size: 10^8,
- reduced search-space size:
  $9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 = 362880$

A simple CLP code I

```
sendmoremoney(Vars) :-
    Vars = [S,E,N,D,M,O,R,Y],
    Vars ins 0..9,
    S #\= 0,
    M #\= 0,
    all_different(Vars),
                  1000*S + 100*E + 10*N + D
    +             1000*M + 100*O + 10*R + E
    #= 10000*M + 1000*O + 100*N + 10*E + Y.

solve(Vars):- Vars=[S,E,N,D,M,O,R,Y],
              sendmoremoney([S,E,N,D,M,O,R,Y]),
              label(Vars).
```

Pretty good results! I

```
?- time(sendmoremoney(V)).
% 6,758 inferences, 0.00 CPU in 0.00 seconds (0% CPU, Infinite Lips
V = [9, _G11470, _G11473, _G11476, 1, 0, _G11485, _G11488],
_G11470 in 4..7,
all_different([_G11470, _G11473, _G11476, _G11485, _G11488, 0, 1, 9
1000*9+91*_G11470+ -90*_G11473+_G11476+ -9000*1+ -900*0+10*_G11485+
_G11473 in 5..8,
_G11476 in 2..8,
_G11485 in 2..8,
_G11488 in 2..8.

?- time(solve(V)).
% 10,088 inferences, 0.01 CPU in 0.00 seconds (308% CPU, 1008800 Li
V = [9, 5, 6, 7, 1, 0, 8, 2].
```

MiniZinc Coding I

```
include "alldifferent.mzn";

var 1..9: S;
var 0..9: E;
var 0..9: N;
var 0..9: D;
var 1..9: M;
var 0..9: O;
var 0..9: R;
var 0..9: Y;

constraint           1000 * S + 100 * E + 10 * N + D
                   + 1000 * M + 100 * O + 10 * R + E
       = 10000 * M + 1000 * O + 100 * N + 10 * E + Y;

constraint alldifferent([S,E,N,D,M,O,R,Y]);
```

MiniZinc Coding II

```
solve satisfy;

output ["   ",show(S),show(E),show(N),show(D),"\n",
        "+  ",show(M),show(O),show(R),show(E),"\n",
        "= ",show(M),show(O),show(N),show(E),show(Y),"\n"];
```